SENSITIVITY INVESTIGATIONS CONCERNING THE STABILITY OF THIN-WALLED SHELL STRUCTURES

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For stability investigations nowadays standard procedures are based on a static approach, when applying the finite element method. Many – slightly different - algorithms can be used to detect and compute so-called stability points, snap-through and bifurcation points. In order to investigate the post-buckling behavior, “unstable” parts of the solution paths are usually computed using e.g. arc-length methods and path switching procedures. For problems with a limited number of degrees of freedom such as beam structures, this is a very successful procedure. However, when investigating shell type structures under compression resp. external pressure the solution path can be rather complex and e.g. multiple branching is possible. To follow the various solution paths, manual as well as automatic control is in principle possible, but extremely inefficient due to the many trial and error actions necessary. Convergence problems in the vicinity of singular points and in the various post-buckling branches, as the condition number of the tangent matrices is often very large, occur rather frequently. In addition, the decision which branch is followed is often rather arbitrary as the nature of the nonlinear solution along this branch is not known a priori. For shells which are not perfectly supported and silo shells with some kind of solid filling contact problems may occur and eigenvalues in static analysis have little meaning for the judgment of stability.

It is particularly questionable, if the post-buckling behavior computed for static loading as described above has any meaning for the real behavior, as buckling is in reality a dynamic process. This implies that the post-buckling behavior should be computed performing a nonlinear transient analysis, see e.g. [1,2], which can be achieved with moderate numerical effort as the effective matrices are better conditioned too.

However, it is not known, which imperfections resp. types of perturbation are leading to the “real” post-buckling path. Thus our focus is on the investigation of the sensitivity of stable equilibrium states along the solution paths using finite perturbations. The sensitivity is hereby defined as the reciprocal value of the minimum kinetic energy that is necessary to transfer the mechanical system from a stable state to an unbounded motion [2]. The major problem involved is the judgment whether the perturbation leads to an unstable or a stable motion either towards a new or towards the old equilibrium state. The Ljapunov characteristic exponents [3, 4] can be taken as a measure which is demonstrated for simple stability problems as well as for realistic shell and silo stability problems. For design purposes also the internal energies are investigated allowing to judge the load carrying capacities.

References

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